

Received May 13, 1773.

XXXVI. *Properties of the Conic Sections; deduced by a compendious Method. Being a Work of the late William Jones, Esq; F. R. S. which he formerly communicated to Mr. John Robertson, Libr. R. S. who now addresses it to the Reverend Nevil Maskelyne, F. R. S. Astronomer Royal.*

S I R,

Read June 24,
1773. **Y**OU well know that the curves formed by the sections of a cone, and therefore called CONIC SECTIONS, have, from the earliest ages of geometry, engaged the attention of mathematicians, on account of their extensive utility in the solution of many problems, which were incapable of being constructed by any possible combination of right lines and circles, the magnitudes used in plane geometry. The properties of these curves are become far more interesting within the two last centuries, since they have been found to be similar to those which are described by the motions of the cœlestial bodies in the Solar system.

Two

Two different methods have been taken by the writers who have treated of their properties; the one, and the more antient, is to deduce them from the properties of the cone itself; the other is to consider the curves, as generated by the constant motion of two or more straight lines moving in a given plane, by certain laws.

There are various methods of generating these curve lines *in plano*; one method will give some properties very easily; but others, with much trouble: while, by another mode of description, some properties may be readily derived, which, by the former, were not so easily come at: so that it appears there may be a manner of describing the curves similar to the Conic Sections, by the motion of lines on a plane, which, in general, shall produce the most essential properties, with the greatest facility.

That excellent mathematician, the late William Jones, Esq; F. R. S. had drawn up some papers on the description of these curves, or lines of the second kind, very different from what he gave in his *Synopsis Palmariorum Matheseos*, published in the year 1706; or from that of any other writer on this subject. A copy of these papers he was pleased to let me take about the year 1740. He had not finished them as he intended; but, in their present state, they appear of too much consequence to be lost; as, it is much to be feared, his own copy, together with many other valuable papers, are; and therefore, I am desirous of preserving them in the Philosophical

Transactions, in the manner I at first transcribed them; although, I am aware, they might have been put into a form more pleasing to the generality of readers: I have indeed annexed larger diagrams than what accompanied the author's copy, in order to render the lines more distinct, as all the relations are to be represented in a single figure, of each kind.

Mr. Jones, having laid down a very simple method of describing these curves, seems to have been desirous of arriving at their properties in as expeditious a way as he could contrive; and therefore he has used the algebraic method, in general, of reducing his equations; and, on some occasions, has used the method of fluxions, to deduce some properties chiefly relating to the tangents; and, by a judicious use of these, he has very much abridged the steps which otherwise he must have taken, to have deduced the very great variety of relations he has obtained: these he intended to have arranged in tables, from whence an equation expressing the relation between any three or more lines of the Conic Sections, might be taken out as readily as a logarithm out of their tables; this he has only partly executed; but it may easily be continued by those who are desirous to have it done, and are sufficiently acquainted with what follows.

From the House of the Royal Society,
April 29, 1773.

THE DESCRIPTION OF LINES OF THE SECOND KIND.

LET the right lines AD , AQ , be drawn on a plane, at any inclination the one with the other. See PLATE XIV. Fig. 1, 2, 3.

In AD , AQ , take Aa , AM , of any given magnitude, and draw MN parallel to AD ,

On the points A , a , let two rulers AP , aP , revolve, and cut MN , AQ , in N and Q , so that AQ be every-where equal to MN .

Then shall the intersection P of the rulers describe lines of the second kind, or curves of the first kind.

Where the right-line Aa , is the first, or transverse diameter.

The point c , bisecting the diameter Aa , is the center.

The right-line PD , drawn parallel to AQ , is the ordinate to the diameter Aa .

The part AD , or CD , of the diameter, is the absciss, when reckoned to begin from A to c , or from c to A .

The right line ab drawn from the center c parallel to the ordinate PD , and terminated in the curve, is called the second, or conjugate diameter.

Those diameters to which the ordinates are perpendicular, are called the axes.

And AM is the parameter to the diameter Aa .

THE PROPERTIES OF LINES OF THE SECOND KIND.

1. Put $Aa=d=2AC=2t$; $Bb=d=2BC=2c$; $AM=2p$; $PD=y$; $CD=x$; $AD=u$.
Then $\overline{PD}^2 = \frac{p}{t} \times ADA$.

Or $yy = \frac{p}{t} \times u \times \overline{d \mp u} = 2pu \mp \frac{p}{t} uu = \frac{p}{t} \times \pm tt \mp xx = \pm pt \mp \frac{p}{t} x^2$.

For $PD = \frac{AM \times AD}{MN} = \frac{AQ \times Da}{Aa}$ (by sim. Δ s). Th. $\overline{PD}^2 = \frac{AM}{Aa} \times ADA$.

2. Consequently $\frac{t}{p} yy = \pm tt \mp xx = du \mp uu = \pm \frac{1}{2} dd \mp xx$.

3. Hence $\frac{t}{p} yy = \mp xx = \overline{t \mp u} \times u = xu$.

4. And $pt = cc$, or $2pd = dd$; for when $y = c = \frac{1}{2}d$, then $x = 0$.

5. Therefore $\frac{\overline{PD}^2}{ADA} = \frac{yy}{\pm tt \mp xx} = \frac{p}{t} = \frac{2p}{d} = \frac{2pd}{dd} = \frac{pt}{tt} = \frac{dd}{dd} = \frac{cc}{tt} = \frac{BC^2}{AC^2}$.

6. The curve line whose property is $yy + \frac{p}{t}uu - 2pu = 0$,
(where the abscissa begins at the curve),
Or $yy + \frac{c^2}{tt}xx - cc = 0$, (where it begins at the center),
is called an Ellipsis. This curve returns into itself. For when $x=0$, then $y=c$;
and when $y=0$, then $x=t$. Which can happen but two ways.

7. The curve line whose property is $yy - \frac{p}{t}uu - 2pu = 0$,

Or $yy - \frac{c^2}{tt}xx + cc = 0$, is called an Hyperbola. This curve spreads out infinitely.
For y increases as x increases; and has four legs tending contrary ways: for xx , or
 yy , may be produced as well from $-x$, or $-y$; as from $+x$, or $+y$.

8. If the point a , is supposed to be at an infinite distance from A , so that a ruler
 AP moves in a parallel position to AD ; then is $yy = 2pu$, or $yy - 2pu = 0$, the property
of the curve described, and is called a Parabola. This curve spreads out infinitely;
for y increases as u increases.

9. Let Aa, Pp , be any two first diameters; Bb, Qq , their second diameters.
 PLATE XIV. Fig. 4.

Draw the ordinates PD, QE , to the diameter Aa , and the ordinate Pd , to the diameter Bb .

Let PT be a tangent, and PM be perpendicular, to the curve, in P ; PT cutting Aa, Bb , produced, in T, t ; and PM , in M, m .

Put the subtangent $DT = s, dt = \sigma$.

Let $AT = r, PM = \pi, CE = w, CT = q = x \pm s = t \pm r$.

Put s, s' , = sine and cosine of the angle MPD , or angle PmB .

R = tabular radius.

Then $CDT = xs = \overline{t \mp u} \times s = \frac{t}{p} yy = \frac{tt}{cc} yy = \pm tt \mp xx = u \times \overline{d \mp u} = ADA$.

For $u \times \overline{t \mp u} = \left(\frac{t}{p} yy = \right) ayj$. Therefore $\left(\frac{u}{y} = \frac{ay}{t \mp u} = \right) \frac{ay}{x} = \frac{s}{y}$ (by sim. Δs).

And $cdt = y\sigma = \frac{p}{s} xx = \frac{cc}{tt} xx = \pm cc \mp yy = Bdb$. For $\left(\frac{u}{y} = \right) \frac{ay}{x} = \frac{x}{\sigma}$.

10. Hence $\overline{AC}^2 = tt = x \times \overline{x \pm s} = x \times \overline{t \pm r} = xq = DCT$.

11. And $\overline{BC}^2 = cc = y \times \overline{y + \sigma} = y \times ct = dct$.

12. Consequently $\frac{\overline{PA}^2}{Bdb} = \frac{xx}{\pm cc \mp yy} = \frac{tt}{cc} = \frac{\overline{AC}^2}{\overline{BC}^2}$.

13. Also $\overline{CE}^2 = ww = \left(\overline{DT}^2 \times \frac{\overline{QE}^2}{\overline{PD}^2} = \overline{DT}^2 \times \frac{AEA}{ADA} = \right) ss \times \frac{tt - ww}{sx} = sx = \frac{tt}{cc} yy = ADA = CDT$.

14. Therefore $AEA = \left(\frac{\overline{CE}^2}{\overline{DT}^2} \times ADA = \frac{sx}{sj} \times sx = \right) xx = \overline{CD}^2$.

15. And $\overline{QE}^2 = \left(\frac{\overline{PD}^2}{ADA} \times AEA = \right) \frac{cc}{tt} xx = \frac{p}{t} xx$.

In the general schemes. PLATE XV. and Fig. 5. PLATE XIV.

16. Let aa , bb , be the longest and shortest axes.

Draw $c\epsilon$ perpendicular to the tangent PT , cutting it in ϵ .

Put $CP=T$; $CQ=C$; $c\epsilon=\epsilon$.

$$\text{Then } c\epsilon = \epsilon = \left(\frac{CT \times CE}{CQ} = \frac{cc}{yy} \times \frac{ty}{cC} \right) \frac{tc}{c} = \frac{AC \times CB}{CQ}.$$

Hence the parallelogram, under the two axes, is equal to the parallelogram under any two diameters.

17. Draw the tangents AN , $a\pi$, to any vertices A , a , meeting any diameters p , q , produced in v , u , and v , u , and the tangent PT in N , n .

$$\text{Then } AV = \left(\frac{CA \times EQ}{CE} \right) \frac{px}{p}. \quad \text{And } AV = \left(\frac{CA \times PD}{CD} \right) \frac{ty}{x}.$$

$$18. \text{ Also } CV = \left(\frac{AV \times CQ}{EQ} \right) \frac{cc}{y}. \quad \text{And } CV = \left(\frac{CP \times CA}{CD} \right) \frac{tT}{x}.$$

$$19. \text{ Hence } PV = (CV \oslash CP) T \times \frac{t \oslash x}{x}. \quad \text{And } pV = T \times \frac{t+x}{x}.$$

$$\text{Also } QU = (CV \oslash CQ) C \times \frac{e \oslash y}{y}. \quad \text{And } qU = C \times \frac{c+y}{y}.$$

20. When Aa and Bb are the longest and shortest axes; and when $y=p$,

Then $xx = tt \mp \frac{t}{p} yy$ will become $tt \mp pt = tt \mp cc$, which call ff .

And $CD = x$, will become $CF = cf = f$.

The points F , f , are called the Focii.

$$21. \text{ Hence } AF = af = \pm t \mp f; \quad Af = aF = t + f.$$

$$22. \text{ Also } \overline{CF}^2 = \overline{cf}^2 = ff = \pm tt \mp ca = \pm tt \mp pt.$$

And in the ellipsis, $\overline{AC}^2 = (\overline{BC}^2 + \overline{CF}^2) = \overline{BF}^2$.

in the hyperbola, $\overline{CF}^2 = (\overline{AC}^2 + \overline{BC}^2) = \overline{BA}^2$.

Hence, a circle described from B , with the distance AC in the ellipsis, or from c , with the distance AB in the hyperbola, will cut the axis Aa in the focii F , f .

23. Draw

23. Draw FP , fP , from the foci F , f , to any point P of the curve; and draw the conjugate diameters Pp , Qq .

Put $PF = z$; $Pf = v$; $PC = T$; $QC = C$; $\frac{f}{t} = \phi$; $\frac{f}{c} = \gamma$. Then

$$\overline{Pf}^2 = vv = yy + xx + 2xf + ff = tt + 2fx + \frac{ff}{tt}xx = TT + ff + 2fx.$$

$$\overline{PF}^2 = zz = yy + xx - 2xf + ff = tt - 2fx + \frac{ff}{tt}xx = TT + ff - 2fx.$$

$$\text{For } yy = \left(\frac{cc}{tt} \times \overline{tt \oslash xx} \right) = \frac{tt \oslash ff}{tt} \times \overline{tt \oslash xx}.$$

$$24. Pf = v = \left(\sqrt{tt + 2fx + \frac{ff}{tt}xx} \right) t + \frac{f}{t}x = t + \phi x = \frac{tt + fx}{t}.$$

$$PF = z = \left(\sqrt{tt - 2fx + \frac{ff}{tt}xx} \right) t - \frac{f}{t}x = t - \phi x = \frac{tt - fx}{t}.$$

$$25. PF \pm Pf = z \pm v = 2t = Aa.$$

$$26. Pf^2 + \overline{PF}^2 = vv + zz = 2yy + 2xx + 2ff = 2tt + 2\phi\phi xx = 2TT + 2ff.$$

$$27. \overline{Pf}^2 - \overline{PF}^2 = vv - zz = 4fx = \overline{v + z} \times \overline{v - z} = 2t \times \overline{v \mp z}.$$

$$28. FPf = zv = z \times \overline{2t \mp z} = 2tz \mp zz = tt \oslash \phi\phi xx = \overline{tt \oslash xx} \times \frac{cc}{tt} xx$$

$$= cc \oslash \frac{ff}{cc} yy = \frac{tt}{cc} yy + \frac{cc}{tt} xx = \overline{CE}^2 + \overline{EQ}^2$$

$$= \overline{CQ}^2 = CC = T \times P.$$

$$29. \text{Let } m = t - z = v - t = \frac{fx}{t} = \phi x = \pm \frac{1}{2}v \mp \frac{1}{2}z$$

$$= \frac{fx}{\sqrt{cc + ff}} = \frac{x}{t} \sqrt{tt \mp cc} = \sqrt{tt \mp zv}$$

$$= \sqrt{tt \mp cc} = \sqrt{ff + cc - zv}.$$

$$30. \text{Hence } \dot{z} = -\dot{v} = \phi \dot{x} = -\frac{f}{t} \dot{x} \pm \frac{fy}{\rho x} j.$$

$$\text{And } \dot{v} = -\dot{z} = \phi \dot{x} = \frac{f}{t} \dot{x} = -\frac{fy}{\rho x} j.$$

$$31. AC = t = \frac{1}{2}x \pm \frac{1}{2}v = \frac{cc}{p} = \sqrt{cc \pm ff} = \frac{fx}{m} = \frac{cx}{\sqrt{cc \omega yy}}$$

$$= \frac{c}{y} \sqrt{sx} = \sqrt{x \times \omega s} = \frac{y^2 + \sqrt{y^2 + 4p^2 x^2}}{2p} = \frac{yy}{2p} + \sqrt{\frac{y^2}{4p^2} + xx}$$

$$32. \overline{CD}^2 = \overline{Pd}^2 = xx = \left[\frac{tyj}{ccs} \right]^2 = \left[\frac{tyj}{ps} \right]^2 = \pm tt \mp \frac{t}{p} yy = \frac{tt}{cc} \times \overline{cc \mp yy}$$

$$= \frac{ff+cc}{cc} \times \overline{cc \mp yy} = tt \mp \frac{tt}{cc} yy = \frac{mm}{\phi\phi} = \frac{tt}{ff} mm$$

$$= \frac{cc+ff}{ff} mm = \frac{tt}{ff} \times \overline{tt \mp zv} = \frac{tt}{ff} \times \overline{tt \mp cc} = \left[\frac{tt}{q} \right]^2$$

$$= \frac{tt}{cc} \times zv - \frac{tt}{cc} yy = \frac{t}{p} \times zv - \frac{t}{p} yy = \frac{tt}{ff} \times \overline{t - z}^2$$

$$= \frac{tt}{ff} \times \overline{v - t}^2 = \left[\frac{tt \mp tz}{f} \right]^2 = \overline{t \mp u}^2 = \sqrt{tt + \frac{1}{4}ss - \frac{1}{2}st}^2$$

$$= \frac{tt}{ff} \times \overline{tt \mp \frac{tt}{cc} \pi\pi} = \frac{t^4}{c^4} \times \frac{ss}{RR}$$

$$33. \text{And } \dot{x} = -\frac{tyj}{px} = -\frac{tyj}{ccx} = -\frac{tyj}{qq} = -\frac{t\dot{z}}{f} = \frac{t\dot{v}}{f}$$

$$34. \overline{PD}^2 = \overline{Cd}^2 = yy = \frac{p}{t} sx = \frac{cc}{tt} sx = \frac{cc}{tt} \times \overline{\pm tt \mp xx} = \pm cc \mp \frac{cc}{tt} xx$$

$$= \pm pt \mp \frac{p}{t} xx = \frac{2cc}{t} u \mp \frac{cc}{tt} uu = \frac{\pm tt \mp ff}{tt} \times \overline{\pm tt \mp xx}$$

$$= \frac{cc}{tt} \times zv - \frac{cc}{tt} xx = \frac{cc}{tt} \times \overline{cc} - \frac{cc}{tt} xx = \frac{cc}{ff} \times \overline{zv - cc}$$

$$= \frac{cc}{ff} nn = \frac{pt}{ff} nn \text{ (putting } nn = zv - cc)$$

$$= \frac{tt \pi\pi - c^4}{ff} = cc \mp \frac{tt}{cc} \times \frac{ss}{RR} = \frac{ss'}{RR}$$

$$35. \text{And } j = -\frac{px\dot{x}}{ty} = -\frac{ccx\dot{x}}{ty} = \frac{px\dot{z}}{fy} = -\frac{px\dot{v}}{fy}$$

$$36. \text{Also } \frac{ff}{cc} yy = zv - cc = \frac{ff}{tt} \times \overline{tt \mp xx} = ff \mp \frac{ff}{tt} xx$$

$$= nn = ff \mp mm = cc - cc.$$

$$37. \overline{PC}^2 = TT = xx + yy = xx \pm cc \mp \frac{cc}{tt}xx = tt - ff + \frac{ff}{tt}xx = cc + \phi\phi xx \\ = cc + tt \omega xv = cc + mm = cc + tt \omega cc \\ = tp + \phi\phi xx = 2cc + ff \omega xv = 2pt + ff \omega xv.$$

$$38. \text{And } \dot{t} = \frac{\phi\phi x \dot{x}}{T} = \frac{\phi\phi x \dot{x}}{\sqrt{tp + \phi\phi xx}} = \left(\frac{\phi\phi x}{T} \times -\frac{tyj}{px} \right) \frac{ffjy}{ptT} = -\frac{ffjy}{ccT}.$$

$$39. \text{Also } \overline{CQ}^2 + \overline{CP}^2 = (cc + tt) = \overline{CB}^2 + \overline{CA}^2.$$

$$40. cT = q = \frac{tt}{x} = \left(tt \times \frac{f}{tm} \right) \frac{ft}{m} = \frac{ft}{t \omega x} = \frac{ft}{\sqrt{tt \omega xv}} \\ ct = \frac{cc}{y} = \frac{cf}{n} = \frac{cf}{\sqrt{xv - cc}}.$$

$$41. \text{And } \dot{q} = -\frac{q \dot{x}}{x} = -\frac{qq \dot{x}}{tt} = \frac{qqjy}{ccx} = \frac{qqjy}{ptx} = -\frac{tt \dot{x}}{xx} \text{ (for } tt = qx).$$

$$42. AT = (\pm CT \mp CA) = \pm \frac{tt}{x} \mp t = \frac{t}{x} \times \pm t \mp x = \pm \frac{ft}{m} \mp t = \pm \frac{ft}{t \mp x} \mp t. \\ aT = (CT + Ca) = \frac{tt}{x} + t = \frac{t}{x} \times t + x = \frac{ft}{m} + t = \frac{ft}{t \omega x} + t.$$

$$43. FT = (\pm CT \mp CF = \pm \frac{ft}{m} \mp f = \frac{\pm ft \mp fm}{m}) = \frac{fx}{t \omega x} = \frac{fx}{m} = \frac{tx}{x}. \\ fT = (CT + Cf) = \frac{ft}{m} + f = \frac{ft + fm}{m} = \frac{fv}{\omega t} = \frac{fv}{m} = \frac{tv}{x}.$$

On Aa describe a circle, draw the tangent TP', and draw CP'.

Continue DP to P', and at right angles to TP', draw FR',

$$\text{Then } FR' = \left(\frac{FT}{CT} \times CP' = \frac{tx}{x} \times t \times \frac{x}{tt} \right) z = FP.$$

$$fr' = \left(\frac{fT}{CT} \times CP' = \frac{tv}{x} \times t \times \frac{x}{tt} \right) v = fP.$$

$$44. \text{DT} = s = \frac{tyy}{px} = \frac{t yy}{ccx} = \frac{\pm it \mp xx}{x} = \frac{t nn}{fm}.$$

$$dt = \sigma = \frac{p xx}{ty} = \frac{ccxx}{t ty} = \frac{cc \mp yy}{y} = \frac{cmm}{fn}.$$

$$45. \text{AN} = \left(\frac{ct}{ct} \times \text{AT} \right) = \frac{cm}{nx} \times \overline{t-x} = \frac{c}{n} \times \overline{f-m}.$$

$$an = \left(\frac{ct}{ct} \times \text{AT} \right) = \frac{cm}{nx} \times \overline{t+x}.$$

$$46. \text{VN} = \left(\frac{\text{PV} \times ct}{pc} = \frac{t-x}{t} \times \frac{t}{x} \times \frac{fc}{n} \right) = \frac{fc}{nx} \times \overline{t-x} = \frac{fc}{nm} \times \overline{f-m}$$

$$= \frac{fc}{\sqrt{zv-cc} \times \sqrt{it-zv}} \times \overline{f - \sqrt{it-zv}}$$

$$\text{UN} = ct = \frac{cc}{y}.$$

$$47. \text{AD} = (\pm \text{AC} \mp \text{CD} = \pm t \mp \frac{tm}{f}) = \frac{t}{f} \times \overline{f \mp m}$$

$$aD = (\text{AC} + \text{CD} = t + \frac{tm}{f}) = \frac{t}{f} \times \overline{f + m}.$$

48. Produce PF, pf, to meet the curve in Π, π;

Draw ΠΔ perpendicular to Λa.

Put FΠ = z', fπ = v', FΔ = x'.

$$\text{Now, FD} = (\pm \text{CF} \mp \text{CD}) = \pm f \mp x = \pm f \mp \frac{tm}{f} = \frac{tz-cc}{f}$$

$$fD = f \mp \frac{tm}{f} = \frac{tv-cc}{f}.$$

$$\text{Then FΔ} = x' = (\text{CA} - \text{CF}) = \frac{tm}{f} - f = \left(\frac{t}{f} \times \overline{t-x} - f \right) = \frac{cc-tz}{f}$$

$$fΔ = f + \frac{tm}{f} = \frac{tv'-cc}{f}.$$

49. From F, f , draw FR, fr , perpendicular to the tangent PT , and cutting it in R, r .

$$FR = \lambda = \frac{FT \times C\phi}{CT} = \frac{cz}{c} = \frac{cz}{\sqrt{zv}} = \frac{cz}{\sqrt{2tz - zz}} = c\sqrt{\frac{z}{v}}$$

$$= c\sqrt{\frac{t - \phi x}{t + \phi x}} = \sqrt{\frac{ptz}{zt - z}} = \frac{ze}{t}$$

$$fr = \lambda' = \frac{fT \times C\phi}{CT} = \frac{cv}{c} = \frac{cv}{\sqrt{zv}} = c\sqrt{\frac{z}{v}}$$

$$= e\sqrt{\frac{t + \phi x}{t - \phi x}} = \sqrt{\frac{ptv}{2t - v}}$$

$$\lambda = \frac{tcz}{v\sqrt{zv}}. \text{ For } \lambda^2 = \frac{c^2 z}{v}. \text{ Th. } 2\lambda\lambda' = \left(\frac{c^2 v z - c^2 z v}{vv}\right) = \frac{2c^2 t z}{vv}$$

$$50. TR = \left(\frac{AT \times RF}{AN} = \frac{tnz}{mc}\right) = \frac{tnz}{fcx} = \frac{tn\sqrt{z}}{fx} = \frac{tn}{m}\sqrt{\frac{z}{v}}$$

$$Tr = \frac{aT \times rf}{an} = \frac{tn\sqrt{v}}{z}$$

51. Draw PM perpendicular to the tangent PT , meeting the axes Aa, Eb , in M, m .

$$DM = \left(\frac{PD \times PD}{TD}\right) = \frac{ccm}{ft} = \frac{cc}{ft} \sqrt{tt \pm zv} = \frac{cc}{tt} x = \frac{p}{t} x$$

$$= \frac{s}{R} \pi = p \mp \frac{p}{t} u = \frac{cc}{tf} \sqrt{tt \mp \frac{tt}{cc} \pi \pi}$$

$$dm = \left(\frac{PD \times Pd}{DM}\right) = \frac{tn}{fc} = \frac{tt}{cc} \sqrt{zv - cc} = \frac{tt}{cc} y = \frac{t}{p} y$$

$$52. CM = (CD \mp DM = x \mp \frac{cc}{tt} x) = \frac{tt \mp cc}{tt} x = \frac{ff}{tt} x = \frac{f}{t} m$$

$$= \frac{f}{t} \sqrt{tt \mp zv} = \frac{f}{t} \times t \mp z = \frac{f}{t} \times \sqrt{tt \mp \frac{tt}{cc} \pi \pi}$$

$$cm = (dm \mp cd = \frac{tt}{cc} y \mp y) = \frac{tt \mp cc}{cc} y = \frac{ff}{cc} y = \frac{f}{c} n = \frac{f}{c} \sqrt{zv - cc}$$

$$53. FM = (CF \cup CM = f \cup \frac{fm}{t}) = \frac{f}{t} \times t \mp m = \frac{fz}{t} = \pm f \mp \frac{ff}{tt} x$$

$$fM = (fc + CM = f + \frac{fm}{t}) = \frac{f}{t} \times t + m = \frac{fv}{t} = f \pm \frac{ff}{tt} x$$

$$54. \overline{Fm}^2 = \overline{fm}^2 = (\overline{cf}^2 + \overline{cm}^2) = ff + \frac{ffnn}{cc} = \frac{ffzv}{cc} = \frac{ff}{cc} \times cc.$$

$$55. AM = (AC \oslash CM) = t \oslash \frac{fm}{t} = \frac{tt \oslash fm}{t} = t \oslash \frac{ff}{tt} x.$$

$$aM = (ac + cm) = t + \frac{fm}{t} = \frac{tt + fm}{t} = t + \frac{ff}{tt} x.$$

$$56. TM = (TF + FM = \frac{fx}{m} + \frac{fx}{t}) = \frac{fxv}{tm} = \frac{zv}{x} = \frac{cc}{x}$$

$$tm = (ct + cm = \frac{fc}{n} + \frac{fn}{c}) = \frac{fxv}{cn} = \frac{zv}{y} = \frac{cc}{y}.$$

$$57. PM = \pi = \left(\frac{PR \times TM}{FT} \right) = \frac{c}{t} \sqrt{zv} = \frac{c}{t} C = \frac{p}{c} C = \frac{c}{t} \sqrt{tt \oslash \phi^2 x^2}$$

$$= \frac{c}{t} \sqrt{t + \phi x \times t - \phi x} = \frac{cc}{tt} \times \frac{R}{s} x = \frac{cR}{ts} \sqrt{cc \mp yy}$$

$$= \frac{c}{tt} \sqrt{t^2 - ffxx} = \frac{c}{t} \sqrt{cc + \gamma\gamma yy} = \frac{1}{t} \sqrt{c^2 + ffyy}$$

$$= \frac{c}{t} \sqrt{2tz \mp zz} = \sqrt{2pz \mp \frac{p}{t} zz}.$$

$$Pm = \pi' = \left(\frac{PM \times Pd}{DM} \right) = \frac{t}{c} \sqrt{zv} = \frac{t}{c} C = \frac{c}{p} C = \frac{t}{c} \sqrt{tt \oslash \phi \phi xx}$$

$$= \frac{t}{c} \sqrt{t + \phi x \times t - qx} = \frac{1}{c} \sqrt{t^2 \oslash ffxx} = \frac{t}{c} \sqrt{cc + \gamma\gamma yy}$$

$$= \frac{t}{cc} \sqrt{c^2 + ffyy} = \frac{t}{c} \sqrt{2tz \mp zz} = \sqrt{\frac{2tt}{p} z \mp \frac{t}{p} zz}.$$

$$58. Mm = (Pm \mp PM = \frac{t}{c} C \mp \frac{c}{t} C) = \frac{tt \mp cc}{ct} C = \frac{ff}{ct} \sqrt{zv}.$$

$$= \frac{ff}{ct} \sqrt{tt \oslash \phi \phi xx}.$$

$$59. PT = \tau = \left(\frac{PD \times PM}{DM} \right) = \frac{n}{m} C = \sqrt{\frac{zv - cc}{tt - zv}} \times zv = \frac{tn}{fx} C.$$

$$= \frac{ty}{cx} C = \frac{ty}{cx} \sqrt{tt \oslash \phi \phi xx} = \frac{\sqrt{zv - cc} \times zv}{t \mp x}.$$

$$Pt = \tau' = \left(\frac{PT \times Pd}{DT} \right) = \frac{m}{n} C = \sqrt{\frac{tt - zv}{zv - cc}} \times vz = \frac{fx}{tn} C$$

$$= \frac{cx}{ty} C = \frac{cx}{ty} \sqrt{tt - \phi \phi xx}.$$

$$60. \text{TN} = \left(\frac{\text{PT} \times \text{TA}}{\text{DT}} = \frac{n c}{m} \times \frac{t}{m} \times \overline{f \omega m} \times \frac{f m}{i n n} \right) = \frac{f c}{m n} \times \overline{f \omega m} = \frac{c c}{x y} \times \overline{t \omega x}$$

$$\text{Tn} = \left(\frac{\text{PT} \times \text{Ta}}{\text{DT}} = \frac{n c}{m} \times \frac{t}{m} \times \overline{f+m} \times \frac{f m}{i n n} \right) = \frac{f c}{m n} \times \overline{f+m} = \frac{c c}{x y} \times \overline{t+x}$$

$$61. \text{PN} = \left(\frac{\text{AD} \times \text{PT}}{\text{DT}} = \frac{t}{f} \times \overline{f \omega m} \times \frac{n c}{m} \times \frac{f m}{i n n} \right) = \frac{c}{n} \times \overline{f \omega m} = \frac{c c}{t y} \times \overline{t \omega x}$$

$$\text{Pn} = \left(\frac{\text{AD} \times \text{PT}}{\text{DT}} = \frac{t}{f} \times \overline{f+m} \times \frac{n c}{m} \times \frac{f m}{i n n} \right) = \frac{c}{n} \times \overline{f+m} = \frac{c c}{t y} \times \overline{t+x}$$

$$62. \text{PR} = \left(\frac{\text{PT} \times \text{FM}}{\text{TM}} \right) = \frac{n z}{c} = n \sqrt{\frac{z}{v}} = \sqrt{\frac{z}{v} \times \overline{z v - c c}} = \frac{f}{c} y \times \sqrt{\frac{z}{v}}$$

$$= \frac{f}{c} y \times \sqrt{\frac{t - \phi x}{t + \phi x}} = \frac{f}{c c} \times \lambda y$$

$$\text{Pr} = \left(\frac{\text{PT} \times \text{FM}_a}{\text{TM}} \right) = \frac{n v}{c} = n \sqrt{\frac{v}{z}} = \sqrt{\frac{v}{z} \times \overline{z v - c c}} = \frac{f}{c} \times y \sqrt{\frac{v}{z}} = \frac{f}{c} y \times \sqrt{\frac{t + \phi x}{t - \phi x}}$$

$$63. \text{T} \xi = \left(\frac{\text{PT} \times \text{TC}}{\text{TM}} \right) = \frac{n t t}{m c} = \frac{t t}{t - z} \sqrt{\frac{z v - c c}{v z}} = \frac{f y t t}{m c c} = \frac{y t^2}{z c c}$$

$$64. \text{P} \xi = (\text{T} \xi \mp \text{TP} = \frac{n t t}{m c} \mp \frac{n c}{m} \times) \frac{n}{m c} \times t t \mp c c = \frac{n}{m c} \times m m = \frac{n m}{c}$$

$$65. \text{T} t = \left(\frac{\text{PT} \times \text{Ct}}{\text{PD}} = \frac{n c}{m} \times \frac{c f}{n} \times \frac{f}{c n} \right) = \frac{f f c}{m n} = f f \sqrt{\frac{z v}{t t - z v} \times \overline{z v - c c}}$$

$$= \frac{t c}{x y} = \frac{t c}{x y} \sqrt{\frac{t t - f f}{t t} \times x x}$$

$$66. \text{Nn} = \left(\frac{\text{PT} \times \text{Aa}}{\text{DT}} \right) = \frac{2 f c}{c n} = \frac{2 c}{y} c = \frac{2 c}{y} \sqrt{z v} = 2 f \sqrt{\frac{z v}{z v - c c}}$$

From c and F, draw cy and F ξ , perpendicular to Pf

Put P ξ = b; Σ = co-sine of the \angle A f P; R = tabular radius..

$$\text{Then } f \gamma = \left(\frac{c f \times f D}{P f} = \frac{t v - c c}{f} \right) = \frac{t v - c c}{u} = \left(\frac{\Sigma}{R} \times c f \right) \frac{\Sigma f}{R}$$

$$\text{And } P f = v = \frac{c c}{t - \frac{\Sigma}{R} f} = \frac{t t - f f}{t - \frac{\Sigma}{R} f}$$

$$\begin{aligned}
 ps = b &= \left(\frac{\pm xx + vv \mp 4ff}{2v} = \frac{\pm TT \mp ff}{v} = \frac{\pm vv \mp vx}{v} \right) \\
 &= \pm \frac{2pt}{v} \mp z = \pm \frac{z+v}{v} p \mp z.
 \end{aligned}$$

Hence $p = \frac{\pm b \mp z}{z+v} \times v$; $v = \frac{pz}{\mp b \pm z - p}$; $z = \frac{p-b}{v-p} \times v$

Therefore p is less than, equal to, or greater than $\pm b \mp z$, in the Ellipsis, Parabola, or Hyperbola.

67. Draw $m\mu$, making the $\angle Pm\mu = \angle PCM$, and cutting PC in μ .

$$\begin{aligned}
 \text{Then } P\mu &= \left(\frac{PM \times Pm}{PC} = \right) \frac{CC}{T} = \frac{zv}{\sqrt{zt \pm cc \mp zv}} = \frac{tt - \phi\phi xx}{\sqrt{cc + \phi\phi xx}} \\
 &= P = \frac{1}{2} \text{ parameter to } PC.
 \end{aligned}$$

68. Let $F\Phi$ be an ordinate to the axis Aa . at the focus F . and ΦG a tangent to the curve in Φ , meeting Aa , BC , AN , an , in G , g , s , s . Then

$$\begin{aligned}
 \Phi\Phi &= \sqrt{\frac{cc}{tt} \times AFa} = \sqrt{\frac{cc}{tt} \times cc} = \frac{cc}{t} = \frac{pt}{t} = p \\
 &= \frac{1}{2} \text{ parameter to } Aa.
 \end{aligned}$$

$$69. CG = \frac{tt}{f} = \frac{ff \pm cc}{f} = \frac{tt}{\sqrt{tt - cc}} = \frac{tt}{\sqrt{t \times t - p}}$$

$$70. FG = (CG \oslash CF = \frac{tt}{f} \oslash f = \frac{tt \oslash ff}{f} = \frac{cc}{f} = \frac{pt}{f}.$$

71. Draw PH parallel to Aa ; and GH perpendicular to Aa , meeting, PH , PT , in H , h ; then

$$PH = DG = (CG \oslash DC = \frac{tt}{f} \oslash x = \frac{t}{f} z = \frac{t}{f} \times PF.$$

$$72. \text{TO} = (\text{CT} \cup \text{CG} = \frac{ft}{m} \cup \frac{tt}{f} = \frac{t}{m} \times \overline{f \cup x} =) \frac{t}{t-x} \times \frac{\pm tx \mp cc}{f} = \frac{tt}{fx} \times \overline{f \cup x}.$$

$$73. \text{Gh} = \left(\frac{\text{FD} \times \text{TO}}{\text{DT}} = \frac{cn}{f} \times \frac{t}{m} \times \overline{f \cup x} \times \frac{fm}{inn} = \frac{c}{n} \times \overline{f \cup x} = \right) \\ \frac{cc}{fy} \times \overline{f \cup x} = \frac{c}{\sqrt{xv-cc}} \times \frac{tx-cc}{f}.$$

$$74. \text{Ph} = \left(\frac{\text{TP} \times \text{DG}}{\text{DT}} = \right) \frac{zc}{n} = \frac{cz}{fy} = z \times \sqrt{\frac{xv}{xv-cc}} = \frac{cz}{fy} \sqrt{xv}.$$

$$75. \text{Rb} = (\text{Ph} - \text{PR} =) \frac{zc}{n} - \frac{zn}{c} = \frac{z}{nc} \times \overline{cc-nn} =) \\ \frac{zcc}{nc} = \frac{zcc}{\sqrt{xv-cc+zv}} = \frac{zc^3}{fy^2}$$

$$76. \text{Fb} = \left(\sqrt{\overline{\text{FR}^2} + \overline{\text{Rb}^2}} = \right) \frac{cz}{n} = \sqrt{\frac{cz}{xv-cc}} = \frac{ccz}{fy} = \frac{ptz}{fy}$$

77. Let $\text{D}\Sigma$ be any Ordinate to the axe Aa , cutting the curve in Σ , and the focal tangent ΦG in σ ;

$$\text{Then } \text{D}\sigma = \left(\frac{\text{F}\Phi + \text{DG}}{\text{FG}} = \frac{cc}{t} \times \frac{tx}{f} \times \frac{f}{cc} = \right) z = \text{FP} = \text{F}\Sigma.$$

78. Therefore $\text{AS} = \text{AF}$; $as = a\text{F}$; $\text{cg} = \text{CA}$; by sim. Δ s.

$$79. \overline{\text{DF}^2} = (\overline{\text{D}\sigma^2} - \overline{\text{D}\Sigma^2} = \overline{\text{D}\sigma + \text{D}\Sigma} \times \overline{\text{D}\sigma - \text{D}\Sigma} =) \text{P}\sigma \times \sigma\text{E}.$$

79. Let the tangents PN , $p\text{L}$, to the opposite vertices P , p , cut the tangents AN , an , to the opposite vertices A , a , in N , n , L , l .

Then $\text{Pn} = p\text{L}$; $an = \text{AL}$; $\text{PN} = pl$; ANal .

For the Trapezia's PCan , $p\text{CAL}$, are similar and equal;

And so are the Trapezia's PCAN , $p\text{cAl}$.

81. $PF \times Mf = \left(\frac{f}{t} \times v z = \right) Pf \times MF = \frac{f}{t} \times FP f.$
82. $CEQ = (xy =) CDF = dCD = DM \times dm.$
83. $AN \times aD = \frac{ctn}{f} = an \times AD = AC \times PD = Dal = DAL.$
84. $RPr = nn = TPg.$
85. $RT r = \left(\frac{nn}{mm} \times tt = \frac{tt-xx}{xx} \times tt = \right) ATa.$
86. $RT f = \left(\frac{tnz}{mc} \times \frac{fv}{m} = \frac{ft}{m} \times \frac{nc}{m} = \right) CTP = rTF.$
87. $NTn = \left(\frac{ffcc}{mmnn} \times \overline{ffommm} = \right) \frac{ffcc}{mm} = PTf.$
88. $PN \times CB = \left(\overline{t \circ x} = \frac{fc}{tn} c = \right) AN \times CQ.$
89. $CAV = \left(t \times \frac{ty}{x} = \frac{tt}{x} \times y = \right) CT \times PD.$
90. $PRb = \left(\frac{cczz}{cc} = \right) \overline{FR}^2.$

Hence Fb is perpendicular to FR .

91. $FMf = \left(\frac{fz}{t} \times \frac{fv}{t} = \right) \frac{ff}{tt} \times zv = \frac{ff}{tt} \times cc = \frac{ff}{tt} \times \overline{CQ}^2$
92. $AMa = \left(tt \circ \frac{ffmm}{tt} = \right) cc \times \frac{ff}{tt} \times zv$
 $= \overline{CB}^2 + \frac{ff}{tt} \times \overline{CQ}^2 = \overline{CB}^2 + FMf.$

$$93. \frac{\overline{PN}^2}{\overline{TN}^2} = \frac{xx}{tt} = \frac{\overline{CD}^2}{\overline{AC}^2} = \frac{\overline{AN}^2}{\overline{VN}^2} = \frac{\overline{CA}^2}{\overline{CT}^2} = \frac{ADa}{ATg} = \frac{ADa}{RTt} = \frac{\overline{AD}^2}{\overline{AT}^2} = \frac{\overline{aD}^2}{\overline{aT}^2} = \frac{CDT}{ATa}$$

$$94. \frac{FT}{CT} = \frac{x}{t} = \frac{FP}{CA} = \frac{FR}{Cg}. \quad \text{Sim. } \Delta s \text{ TFR, } TCg.$$

$$\text{And } \frac{AD}{DT} = \frac{x}{t+x} = \frac{CD}{AD} = \frac{AC}{AT}$$

$$95. \frac{fT}{CT} = \frac{v}{t} = \frac{fP}{AC} = \frac{fy}{Cg}. \quad \text{And } \frac{AT}{DT} = \frac{t}{t+x} = \frac{AC}{AD} = \frac{CT}{AT}$$

$$96. \frac{PM}{Pm} = \frac{cc}{tt} = \frac{p}{t} = \frac{\overline{BC}^2}{\overline{AC}^2}. \quad \text{And } \frac{Pm}{fm} = \frac{t}{f} = \frac{AC}{CF}$$

$$97. \frac{FMf}{PM^2} = \frac{ff}{cc} = \frac{t+c \times t-c}{cc} = \left(\frac{tt-t^2}{t^2} \right) = \frac{t-p}{p} = \frac{\overline{CF}^2}{\overline{CB}^2}$$

98. Let PM , the perpendicular to the tangent PT , cut the axis Aa in M ; and fP , produced, cut FR in ϕ .

Then will PM bisect the angle FPf .

For $PF \times Mf = Pf \times MF$.

99. And the angle FPT is equal to the angle fPt .

For $\angle TPF + \angle FPM = \angle tPf + \angle fPM$.

100. Therefore PT will bisect the angle $FP\phi$.

For $\angle \phi PT = (\angle fPt =) \angle FPT$.

101. Consequently $P\phi = PF$. And $R\phi = RF$.

102. A circumference of a circle described from t , with the radius tN , will cut the axis Aa in the foci F, f .

For $TN \times Tn = TF \times Tf$.

103. A circumference of a circle described from c , with the radius cA , will cut the tangent PT in R , r .

Whence the perpendiculars RF , rf , to that tangent, will cut the axis Aa in the focii F , f .

$$\text{For } TR \times Tr = TA \times Ta.$$

104. A circumference described from B , with AC , in the Ellipsis, or from c , with AB in the Hyperbola, will cut the axis Aa , in the focii F , f .

$$\text{For, in the Ellipsis, } tt = cc + ff, \text{ or } \overline{AC}^2 = (\overline{BC}^2 + \overline{CF}^2) = \overline{BF}^2.$$

$$\text{And in the Hyperbola, } ff = tt + cc, \text{ or } \overline{CF}^2 = (\overline{AC}^2 + \overline{BC}^2) = \overline{AB}^2.$$

105. Let cQ produced, cut PF , Pf , in z , x ; draw mz , mx , and MZ , MX , parallel to them, cutting PF , Pf in z , x .

$$\text{Then } Px = CR = CA = Pz = t.$$

106. Hence $\angle Pxz = \angle Pzx$; and $\angle mxz = \angle mzx$.

For Pm is perpendicular to zx .

Consequently, the angles Pzm , Pxm ; PZM , PXM are equal.

107. And the triangles Pzm , Pxm , are similar and equal:

And so are the triangles PZM , PXM .

Consequently, the trapezias $Pzmx$, $PZMX$, are similar.

108. Let CR , cr , cut PF , Pf , in K , k .

$$\text{Then } cK = \left(\frac{CF \times Pf}{Ff} \right) = \frac{1}{2} Pf = Pk = kr.$$

$$\text{And } ck = \left(\frac{cf \times PF}{Ff} \right) = \frac{1}{2} PF = PK = KR.$$

109. Also $Px = Pz = \left(\frac{PM}{Pm} \times Pz = \frac{cC}{t} \times \frac{c}{tC} \times t = \right) \frac{cc}{t} = \left(\frac{pt}{t} \right) = p$.

110. The Trapezias $FGbR$, $PDFR$, $frtC$, are similar, and consequently their corresponding parts are proportional.

$$\text{That is, } \left\{ \begin{array}{l} \frac{FG}{PD} = \frac{Gb}{DF} = \frac{bR}{FR} = \frac{RF}{RP} \\ \frac{fr}{r} = \frac{rt}{t} = \frac{tC}{c} = \frac{cf}{f} \end{array} \right\}$$

For the triangles RFb , RPF , and FbG , FPD , are similar.

111. The Trapezias $CgPD$, $tCMP$ are similar, and consequently their corresponding parts are proportional.

$$\text{That is, } \frac{Cg}{tC} = \frac{gP}{CM} = \frac{PD}{PM} = \frac{DC}{Pt}$$

112. And CR , cr , are parallel to Pf , PF , and equal to CA .

$$\text{For } RTf = CTP = rTF.$$

113. Let Σ = sine of the $\angle tPf$, or TPF ; R = tabular radius.

$$\text{Then } \frac{R}{\Sigma} = \left(\frac{Pf}{fr} \right) = \frac{c}{c} = \frac{\sqrt{zw}}{c}.$$

Put Σ = sine of the $\angle PCQ$, made by any diameter and its ordinate.

$$\text{Then } \frac{TT}{cc} = \frac{1}{2}ff \pm \sqrt{\frac{1}{4}f^4 \omega \frac{r^2 t^2 c^2}{\Sigma^2}}; \quad \frac{tt}{cc} = \frac{1}{2}FF \pm \sqrt{\frac{1}{4}F^4 \omega \frac{T^2 c^2 \Sigma^2}{rr}}.$$

114. Let the parallels $Pf\Pi$, $c\beta r$, $pf\pi$ be drawn, cutting the curve in Π , β , π ; and ordinately applied to some diameter (2τ), whose parameter is 2π , and semi-conjugate $c\beta = x$, to which $P\pi$ is ordinately applied at δ .

$$\text{Then } c\delta = \left(\frac{PF + \pi f}{2} = \frac{PF + f\Pi}{2} \right) = \frac{1}{2}z + \frac{1}{2}z' = \frac{1}{2}P\Pi = \left(\frac{c\beta^2}{Cr} \right) = \frac{c\beta^2}{AC} = \frac{x\pi}{t} = \frac{\pi\tau}{t}.$$

$$115. \overline{c\beta^2} = \left(\frac{1}{2}AC \times P\Pi \right) = \frac{1}{2}t \times \overline{z + z'} = \left(AC^2 \times \frac{P\Pi}{APa} \right) = \frac{tt}{cc} z z' = \frac{t}{p} z z'.$$

$$116. \frac{FP}{P\Pi} = \frac{z}{z'} = \frac{RD}{F\Delta} = \frac{tz - cc}{cc - tz'} = \frac{z - p}{p - z'}.$$

$$117. P\Pi = \frac{pzx}{2z - p} = \frac{1}{2}p \times \overline{z + z'} = \frac{1}{2}p \times P\Pi.$$

$$118. P\Pi = z + z' = \frac{2zx}{2z - p} = \frac{2\pi\tau}{t} = \frac{\tau}{t} \times 2\pi.$$

$$\begin{aligned} \overline{CB}^2 &= cc = pt = \pm tt \mp ff = \pm t \mp f \times t + f = Afa = VAU = tcd = CFG \\ &= PM \times Cg = FR \times fr = AN \times an = al \times AL = AC \times F\Phi = tt \times \frac{PM}{PM} \\ &= tt \times \frac{DM}{DC} = tt \times \frac{dC}{dm} = tt \times \frac{PM^2}{CQ^2} = tt \times \frac{PD^2}{ADa} = tt \times \frac{PD^2}{TDC} = ff \times \frac{PD}{Cm} \\ &= \left[\frac{CQ \times Cg}{AC} \right]^2 = \left[\frac{AN \times CQ}{PN} \right]^2 = AMa - FMf = FPf - RPr = ff \times \frac{t}{t-p} \\ &= TT + CC - tt = \frac{tt}{xx} \times zvu + xx - tt = \frac{1}{2}zv \pm \sqrt{\frac{1}{4}z^2v^2 + ffyy}. \end{aligned}$$

$$\begin{aligned} \overline{CQ}^2 &= CC = P \times T = MPm = NPn = TPt = FPf = CP\eta = Lpl \\ &= PN \times pL = CD \times TM = PD \times tm = \frac{tt}{cc} \times \overline{PM}^2 = \frac{cc}{tt} \times \overline{Pm}^2 = \frac{cc}{ff} \times \overline{fm}^2 \\ &= \left[\frac{tt}{ff} \times FMf = \frac{ct}{ff} \times Mm \right]^2 = \left[\frac{ACB}{Cg} \right]^2 = \overline{PM}^2 + FMf = \overline{BC}^2 + RPr \\ &= tt + cc - TT = tt - \frac{ff}{tt}xx = \frac{tt}{cc}yy + \frac{cc}{tt}xx = \frac{DC}{DT} \times \overline{PT}^2. \end{aligned}$$

$$\begin{aligned} \overline{CF}^2 &= ff = tt - cc = tt - tp = \frac{cc}{p} \times t - p = \frac{tt}{xx} \times mm = P\epsilon \times Tt \\ &= \frac{tt}{cc} \times FMf = \frac{cc}{cc} \times \overline{fm}^2 = \frac{mm}{cc} \times FTf = \frac{mm}{cc} \times NTn. \end{aligned}$$

The semi-parameter (p) to the greater axis (Aa) is equal to

$$\begin{aligned} F\Phi &= PZ = \frac{Afa}{AC} = \frac{VAU}{AC} = \frac{CFG}{AC} = \frac{tcd}{AC} = \frac{PF\Pi}{\frac{1}{2}P\Pi} = \frac{PM \times Cg}{AC} \\ &= \frac{FR \times fr}{AC} = \frac{AN \times an}{AC} = \frac{AC \times PM}{Pm} = \frac{AC \times DM}{DC} = \frac{AC \times dC}{dm} = \frac{BC \times PM}{CQ} \\ &= \frac{BC \times CQ}{Pm} = \frac{\overline{BC}^2}{AC} = \frac{cc}{t} = \frac{t\yy}{sx} = \frac{\pm tt \mp ff}{t} = \frac{cc - xy}{xx} \times t \\ &= \frac{CC + TT - tt}{t} = \frac{CC + xx - tt}{xx} \times t = \frac{2t - z}{tz} \times \overline{FR}^2 = \frac{2t - v}{tv} \times \overline{fr}^2. \end{aligned}$$

The semi-parameter (p) to any diameter (Pp) is equal to

$$\begin{aligned} P\eta &= \frac{CQ^2}{PC} = \frac{CG}{T} = \frac{MPm}{PC} = \frac{NPn}{PC} = \frac{TPt}{PC} = \frac{FPf}{PC} = \frac{Lpl}{PC} = \frac{PN \times pL}{PC} \\ &= \frac{CD \times TM}{PC} = \frac{PD \times tm}{PC} = \frac{PT \times PN}{PV} = \frac{tt}{cc} \times \frac{\overline{PM}^2}{PC} = \frac{cc}{tt} \times \frac{Pm^2}{PC} \\ &= \frac{cc}{ff} \times \frac{\overline{fm}^2}{PC} = \frac{tt}{ff} \times \frac{PMf}{PC} = \frac{tt + cc - TT}{T}. \quad [\text{See TAB. XIV, XV.}] \end{aligned}$$

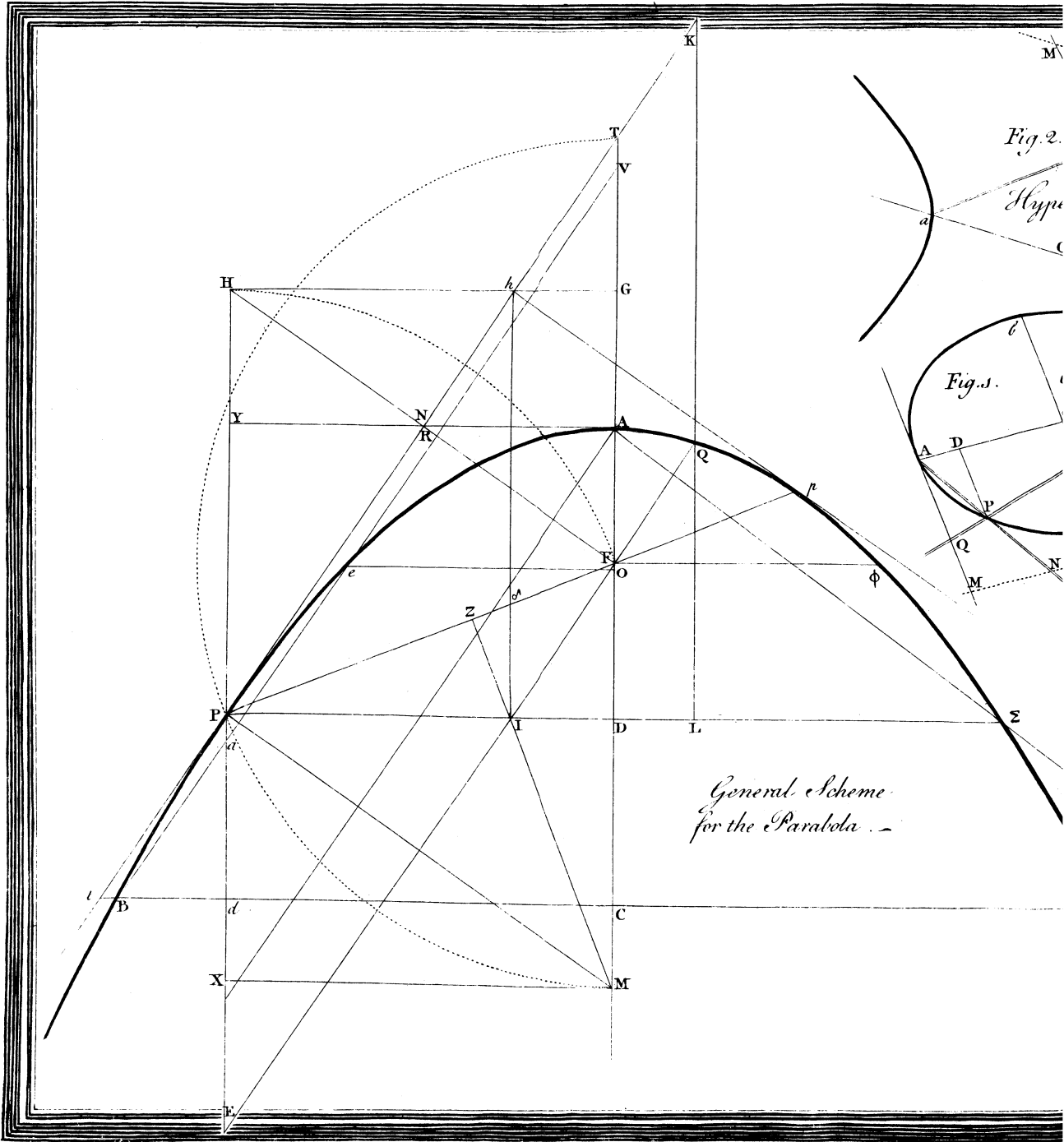


Fig. 2.

Hyperbola.

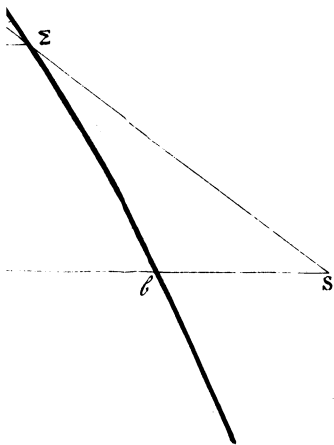
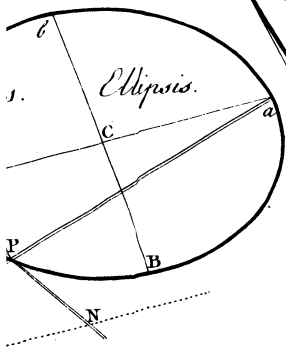
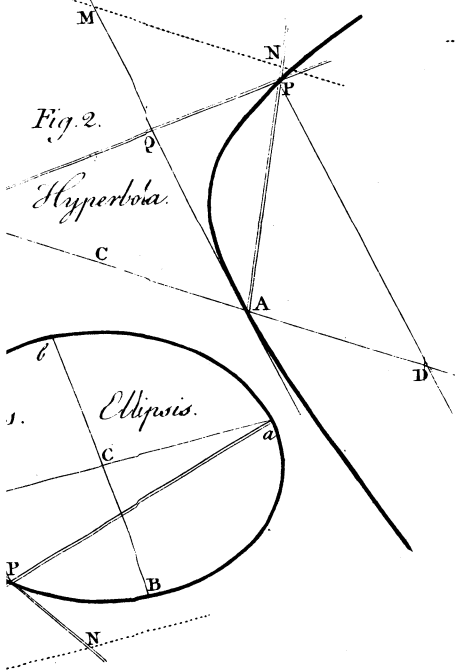


Fig. 3.

Parabola.

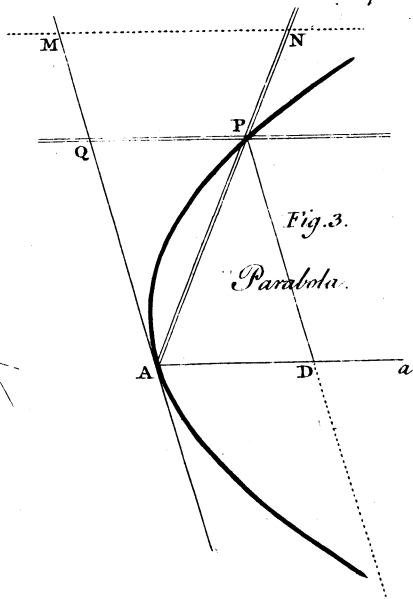
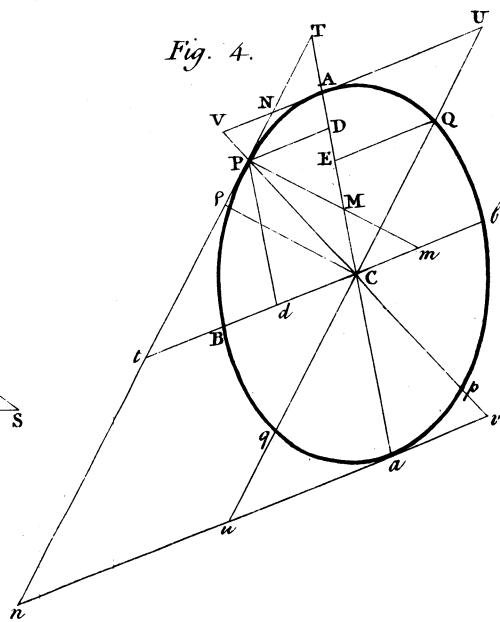
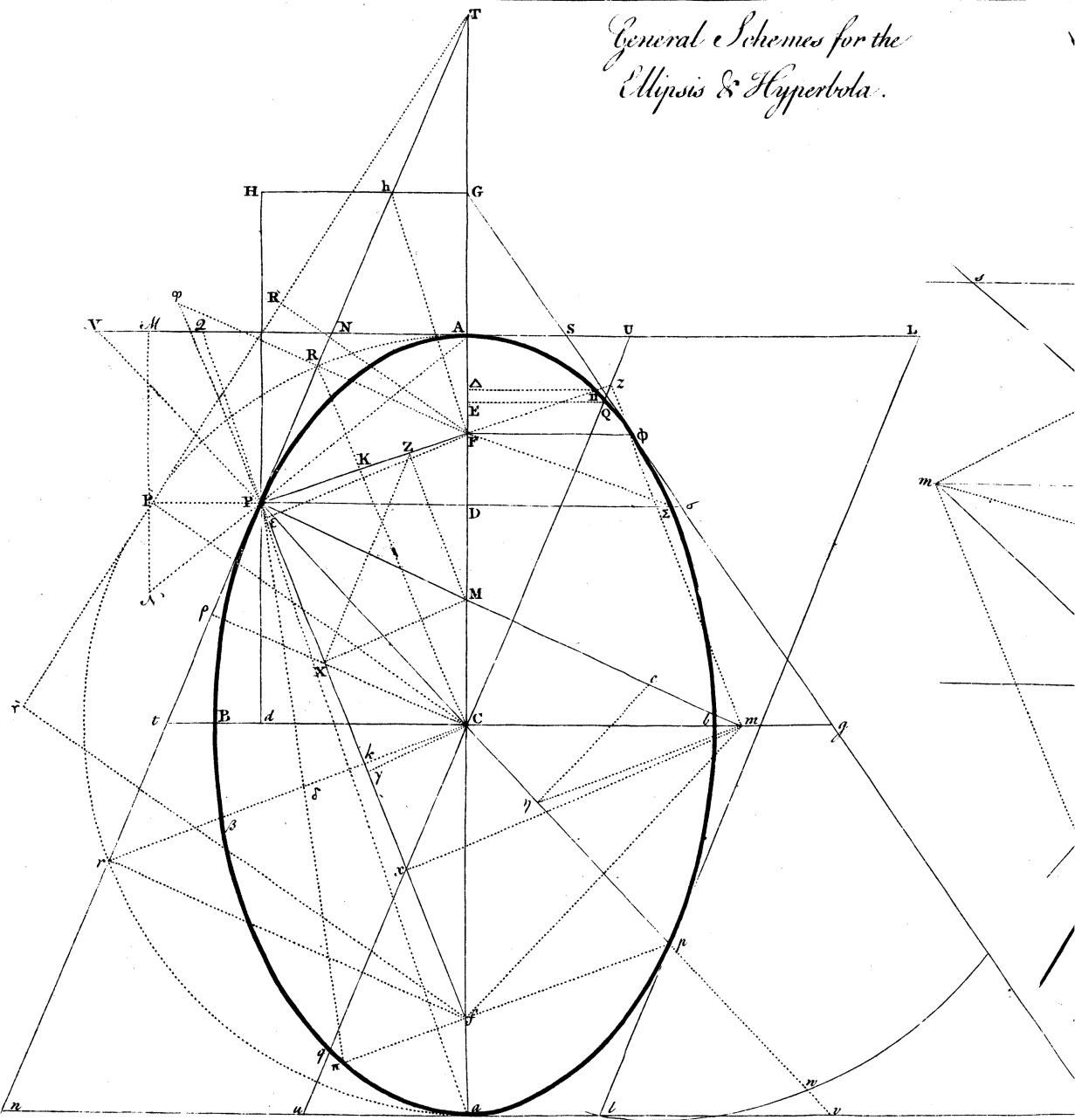


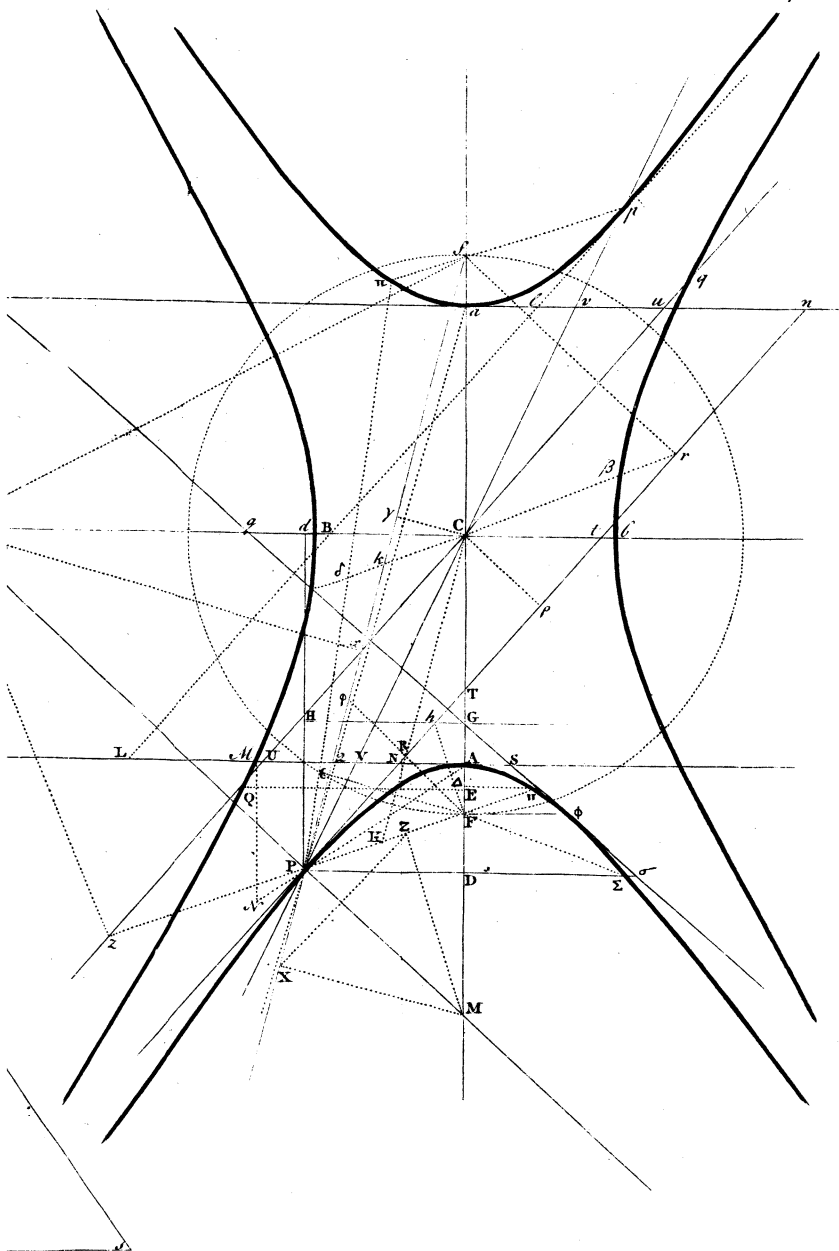
Fig. 4.



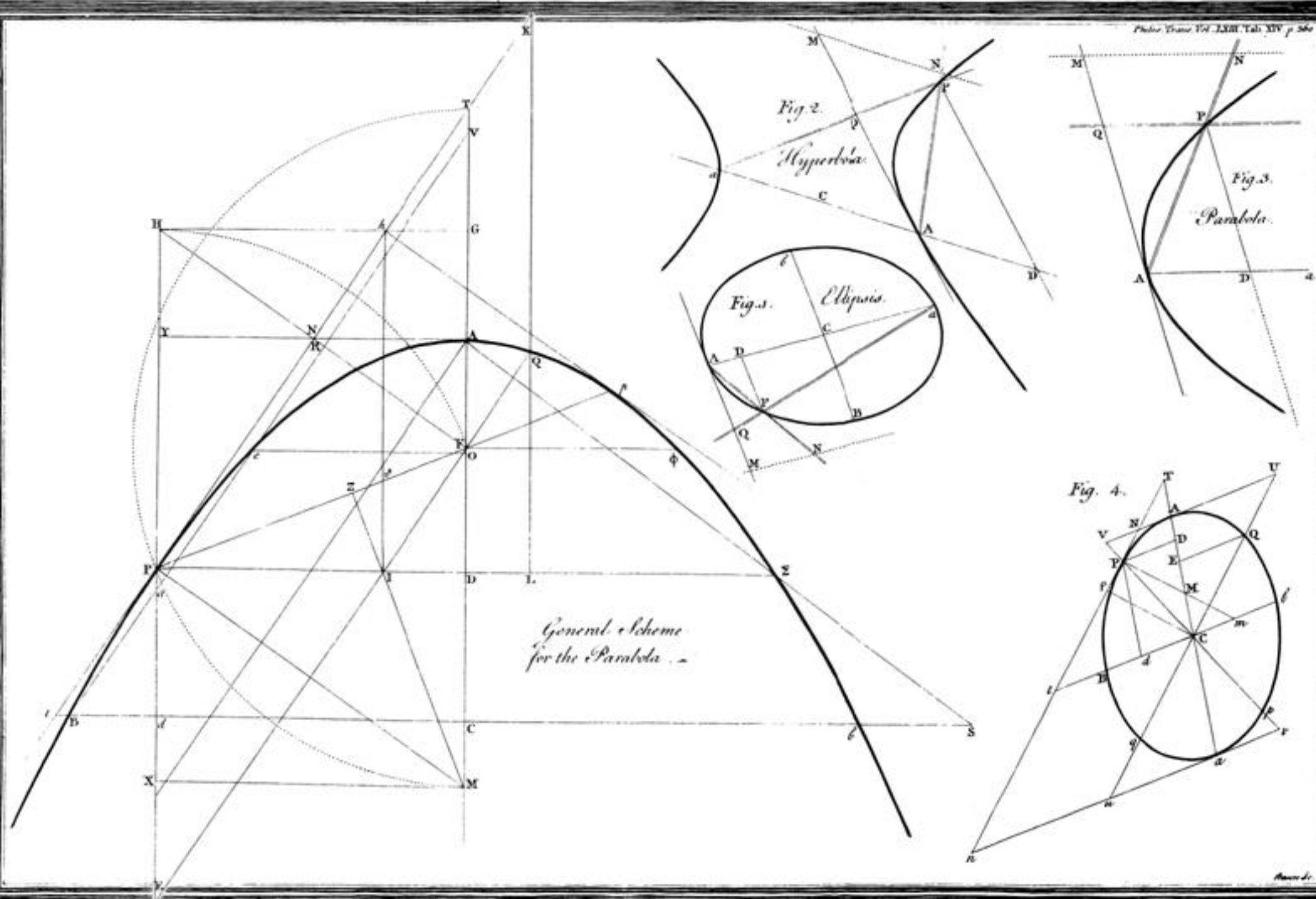
Barrow sc.

*General Schemes for the
Ellipsis & Hyperbola.*





Waire de



General Scheme for the Parabola.

General Schemes for the
Ellipsis & Hyperbola.

